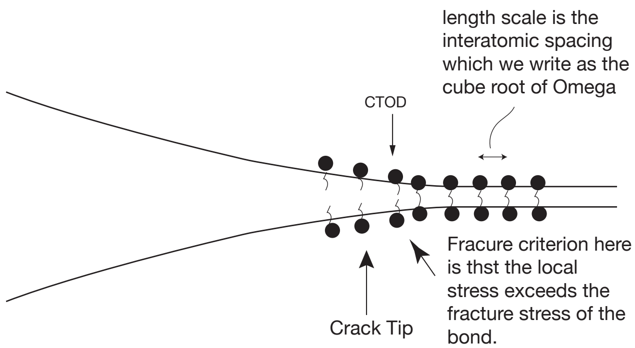
03D_At the Crack Tip

Summary:

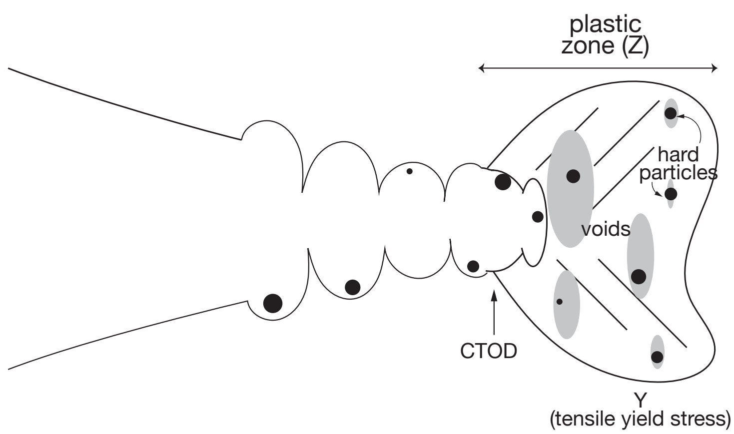
A crack propagates by rupture at the crack tip. While the work of fracture, $2\gamma_F$, is the work done for crack advance (expressed as work done per unit area) which can be analyzed in terms of the work done by the environment (the change in potential and the stored elastic energy), that leaves open the local mechanism that can account for the magnitude of $2\gamma_F$. This energetic approach does not consider the materials science aspects of crack propagation.

In this lecture we will enforce a local criterion for fracture at the crack tip for brittle and semi-brittle fracture by a materials science. These local criteria are expressed by

(i) The local tensile stress experienced at the crack tip



(ii) The crack tip opening displacement (CTOD)



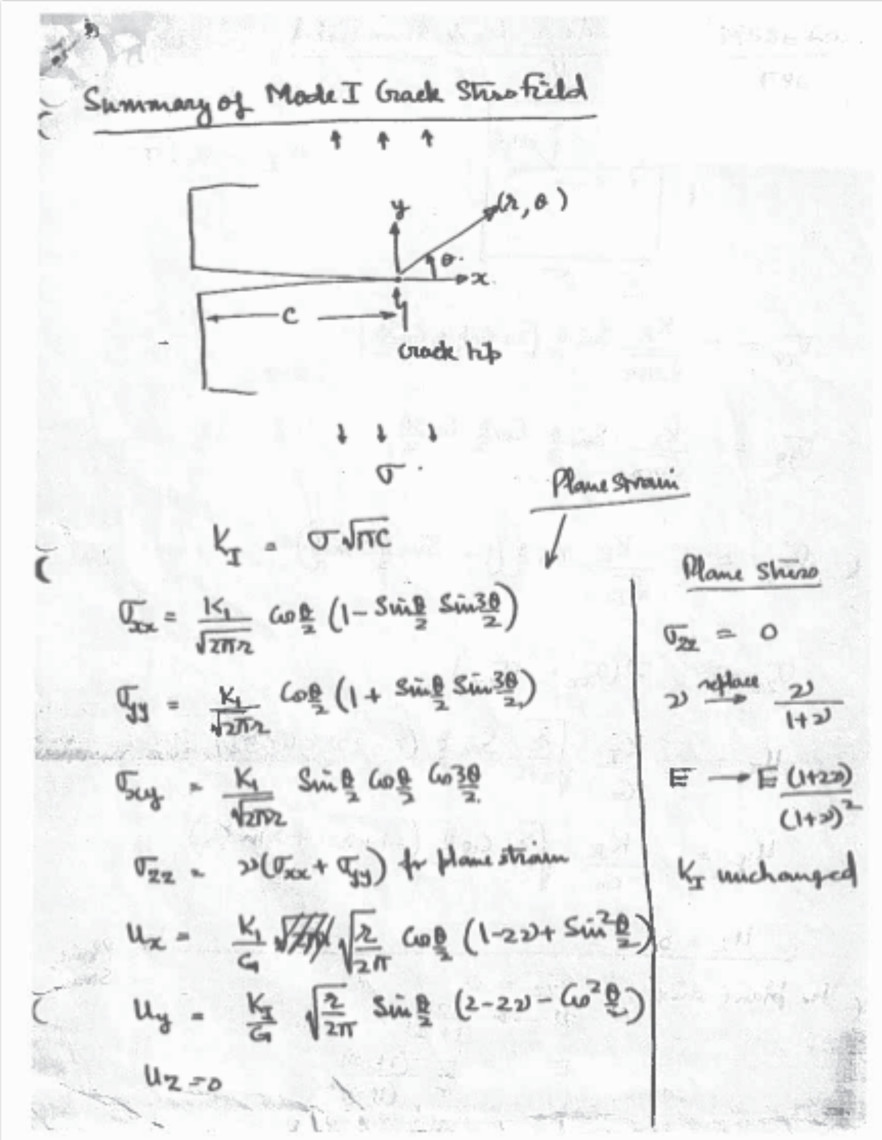
The crack must open to a certain extent to accommodate the damage at the crack tip so that the crack can advance.

To analyze these two mechanisms of fracture at the crack tip, we need to know the stresses and the displacements the occur in response to an applied stress.

The stresses and displacements at the crack tip can be described in terms of the stress intensity factor

$$K_I \approx \sigma\sqrt{c}$$

where σ is the tensile stress normal to the plane of the crack, and c is the crack size (like a diameter of a penny)



Summary of Mode I Crack Stress field

(crack tip, (r, θ), c, σ)

$K_I = \sigma\sqrt{\pi c}$

$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$

$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$

$\sigma_{xy} = \frac{K_I}{\sqrt{2\pi r}} \sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$

$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy})$ for Plane strain

$u_x = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left(1 - 2\nu + \sin^2\frac{\theta}{2}\right)$

$u_y = \frac{K_I}{G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left(2 - 2\nu - \cos^2\frac{\theta}{2}\right)$

$u_z = 0$

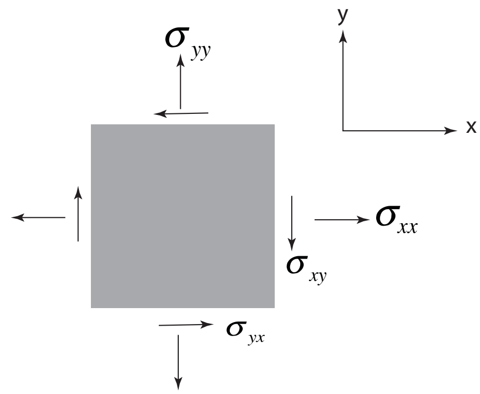
Plane Strain → Plane Stress

$\sigma_{zz} = 0$

$\nu \xrightarrow{\text{Plane}} \frac{\nu}{1+\nu}$

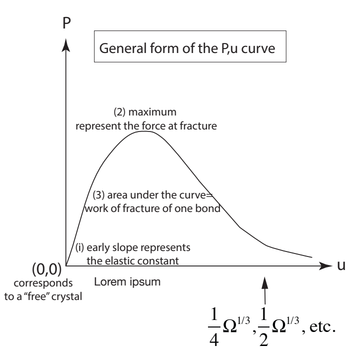
$E \rightarrow E\frac{(1+2\nu)}{(1+\nu)^2}$

K_I unchanged



Consider the criterion for brittle fracture

The stress required to break bonds from the spring model



The length scale for "u" is related to the interatomic spacing, that is to $\Omega^{1/3}$

$$\frac{F_o}{\Omega^{2/3}} = \sigma_{\max} = \frac{E}{2\pi}$$

The fracture criterion is that the tensile force at the first bond from the crack tip must exceed F_o, or the stress at that bond must exceed $\frac{F_o}{\Omega^{2/3}}$ since $\Omega^{2/3}$ is the foot print of one bond.

In terms of the stresses in front of the crack tip the fracture criterion becomes,

$$\sigma_{yy}(r=\Omega^{1/3},\theta=0)=\frac{E}{2\pi}$$

from the equations given above we have that

$$\sigma_{yy}(r=\Omega^{1/3},\theta=0)=\frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}}$$

Therefore we get a final result that

$$\frac{K_{IC}}{\sqrt{2\pi\Omega^{1/3}}}=\frac{E}{2\pi}$$

$$K_{IC}=\frac{E\sqrt{\Omega^{1/3}}}{\sqrt{2\pi}} \qquad (1)$$

Eq. (1) combines mechanics (the derivation of stresses etc. in terms of the stress intensity factor) with materials science that related the bond strength to the elastic modulus.

For silica glass:

Silica Glass		SiO2
E	73	GPa
	73000000000	Pa
Mol wt	60	g/mole
Density	2.65	g/cm^3
molar vol	22.64150943	cm^3
	2.26415E-05	m^3
N_A	6.02E+23	/mol
Omega	3.76E-29	m^3
"a"	3.35E-10	m
Numerato	1.34E+06	
Denom	2.506391829	
K_IC	5.33E+05	Pa m^1/2
	5.33E-01	MPa m^1/2

This value ~0.5 MPa $m^{1/2}$ is in very good agreement with the data for silica glass given in the "fracture map".